A real-estate bubble model

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Abstract

We investigate the potential effect that client feedback and several cognitive biases may cause in the real estate valuation process. The departures from the normative appraisal model are being viewed in the light of the cyclical nature of the US real estate market. Cognitive biases present in the positive valuation models may exacerbate the already documented methodological biases caused by oversimplifying assumptions. If this is true, then a further explanation may be sketched for the existing US real estate market bubble.

Key words: behavioral real estate, cognitive bias, client feedback, real estate bubble.

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1 Introduction

In the beginning of the 1990's Diaz ([8], [9])developed a new research paradigm in real estate by considering the human characteristics of the real estate appraisers as one of the central determinants in the property valuation process. The decision-making process garnished with a variety of cognitive biases provided a rich source of evidence supporting various inconsistencies observed previously in the literature [13], [20]. Diaz's work gave the start to a whole strand of literature which identifies clear differences between the normative and the positive valuation methodology.

Biases observed in general heuristics [18] are documented also to occur in the process of professional real estate appraising. The importance of this evidence has a tremendous impact on the beneficiaries of the appraising service. In the residential real-estate market, the purchase of a house is usually the largest investment made by the average individual or family. The potential loss caused by an incorrect evaluation will be supported by either the agent buying the house for a price higher than its real value or by the bank holding in its portfolio a mispriced asset (in an unfortunate market situation, both the bank and the mortgaging agent might be hurt).

In the United States, the rules of real estate professional appraisal are set by the Appraisal Standards Board (which is an institution chartered by the US Congress) and are all gathered in a document entitled the Uniform Standards of Professional Appraisal Practice (USPAP). This document represents the recommended evaluation methodology (the normative evaluation methodology) to be followed by any certified US appraiser. It is a set of indications and rules of conduct that define the way the assessment process should be carried out. The USPAP evaluation guidelines are cognitively-demanding and require the valuer to start with a broad view on the general state of the market analyzing an entire set of market parameters and finish by focusing on the subject property. What actually happens in practice is that the appraisers takes in consideration information pertaining mostly to the subject property in question, disregarding the rest of the informational ques and focuses on inflation as one of the main market determinants of the expected house price [2], [3]. This fact will be explicitly modeled in the bubble model described in section 3.

The first bias we focus on is the client feedback bias as defined by Wolverton et. al in [21]. Its impact and spread over international markets has been already well documented in [7]. Basically, this bias indicates that the appraisers' initial task as impartial valuers became more of a confirmation task with the purpose of satisfying the demands of mortgage institutions trying to survive in an increasingly competitive environment [11]. For the lender to make business he needs a valuation from the appraiser which justifies the loan even if sometime this is done at the expense of an increased risk. The increased risk comes from a collateral which does not fully justify the loan in terms of market value and that may translate in negative equity and an increased mortgage-to-income ratio. This becomes ultimately nocive in case of an economic downturn: increased unemployment and a shrinking income share available for the mortgage payments may lead to an avalanche of foreclosures. This kind of lender pressure is similar in spirit to the pressure that previously was exerted on corporate security analysts issuing sell recommendations. The companies that were on the sell list, as a punitive measure, banned these analysts from their informative corporate meetings and refused to collaborate with them [16]. Although we do not want to argue that this is a determinant of the real estate bubble, it is indicative that the same unethical actions that inflated the .com bubble are present also in the real estate market, with potentially the same unhealthy effects. It is not a coincidence but rather a common feature. The importance of transparency has been recognized after the bust in 2001 when regulators required companies to allow all experts to participate to their informative meetings.

The second bias on which we focus is the anchor and adjustment bias [18]. When this is present, it causes the problem-solver to anchor to a certain value he considers important and then adjust his problem estimate to this value in order to obtain a solution to his problem. This bias interweaves itself with the need for efficiency and speed of the evaluation practice. Efficiency and speed are two key traits that make the difference in the appraisal process. Efficiency is attained when developing a heuristic that allows the appraiser to bypass the cumbersome requirements of the recommended valuation methodology while speed comes from reusing previously acquired information and recycling it so as to fit the requirements of a new evaluation task. During this information processing, the appraiser also exhibits a recency bias - he incorporates in his analysis mostly information recently acquired and already processed. When the above ingredients combine, the result is an actual evaluation methodology (the positive evaluation methodology) that clearly departs the USPAP. The reason for this departure is that the appraiser will anchor on previous value estimates (these obviously will be the first to come to his head due to the recency bias) and adjust subsequent appraised values to these levels. He will disregard cognitively demanding market data by incorporating in his analysis inflation as the determinant market parameter plus a few key characteristics of the submarket to which the subject property belongs to. All these come to explain the observed appraisal smoothing [13], [20].

Of just as much importance to our study is the so called upward revising bias. The upward revising bias is a mental inclination towards increasing evaluation values when the appraiser observes that his previous estimates were below actual marker prices. This bias (along with the focus on inflation) will be the major modeling ideas when simulating the appraiser's effect on the actual market price. More explicitly, this bias refers to the fact that when given evidence that the evaluated value for a given property is too low, most appraisers and student-appraisers will adjust the value upwards yet when given feedback that the value is too high they will not adjust the value downwards in the same way and to the same extent [14], [15]. Therefore, already from the early years of their studies and apprenticeships, appraiser-trainees have the tendency to overestimate property values when offered feedback of values being too low. When adult, they will also face the reality of coercive feedback and environmental perception feedback [21] - the diplomatical names describing lender pressure. This is a very important feature to model because if in a market, the percentage of transactions using appraisal services is high then this bias will eventually impact the actual asset price by increasing it over the fundamental values.

2 The real-estate market

The biases mentioned in the introduction cannot alone lead to a drastic market disequilibrium in which prices increase far over historic averages. A certain state of the market must occur such that the above mentioned cognitive inclinations amplify an existing tendency of increased prices. A close look at the real estate market reveals a cyclical nature of both demand and offer both in the renting and in the purchasing sector [4],[17]. Looking at the 120-city Index measuring dollar adjusted spending in construction we discover a roughly 19-year long cycle in the construction sector [5] which repeats itself with extreme regularity from the beginning of the century. Extending this cycle to our days we arrive at a peak around the end of the 1990's (around 1998-1999). If we presume that there was some pressure already back then from the mortgage lenders and that appraisers did indeed anchor on the late 1990's appraised values we might presume that the beginning-of-2000 prices instead of dropping stayed at the same high levels as dictated by the cycle's peak. To support this claim we mention also the fact that the dominating pricing technique in the real-estate market is the DCF analysis using trend-driven market parameters.

Brown [5] offers a clear investigation on the impact of trend-driven DCF valuations as compared to cycle-driven DCF models: the trend-driven model will overestimate cash flows near the peak and underestimate them at the through. In our case, this fact not only kept the prices at levels pertaining to a real estate in peak but pushed them forward. At the same time, the Federal Reserve influenced further more the market by lowering the interest rates right after the market crash in 2001. The aim of the measure was to ease the impact of the crash and to limit the shock waves spreading throughout the economy (Alan Greenspan 2003) [1]. Belsky offers proof that the effect of the this measure was an increase in personal consumption due to an overall wealth effect caused by the housing market either directly through realized capital gains (house prices rose) or indirectly through home equity borrowing (more people started to "extract" equity from their homes through second mortgages or lines of credit). Therefore around the beginning of the year 2000 the real-estate market was at a peak and was kept at a peak due to the oversimplifying assumptions used in the DCF evaluations of real-estate developers. No later than one year after this moment, the stock bubble bursts and the Federal Reserve decides to dampen its impact through lower interest rates which opened the appetite for consumption and home purchasing. The measure of the Federal Reserve was perhaps the trigger of the snow-ball effect of increasing home prices.

One important fact to notice from Belsky's work is the difference between the housing-related wealth effects on consumer spending as compared to the stock market effects. He finds that in the long run consumers spend around 5.5 cents for every dollar gain in both real estate and corporate equity wealth, *but* the consumption from housing wealth approaches the 5.5 level a lot faster than the consumption caused by an increase from equity wealth. Within one year from the increase in the housing wealth, roughly 80% of the long-run wealth effect (4.5 cents) is assimilated in consumption whereas it takes about five years for the 4.5 cents increase to be visible when there is a corporate stock wealth increase. Thus an increase in home prices will increase consumption a lot faster than a stock price increase making the real-estate market an even more attractive investment field. Case et al. (2001) [6] as well as Edison and Slok (2001) [10] indicate that one possible reason is the difference in the volatility of the two asset classes, namely real estate versus corporate stocks. The lower the volatility of an asset, the more permanent the wealth effect is perceived by the consumer thus higher the propensity to increase consumption.

Wolverton [21] indicates that a large fraction of the existing certified appraisers are working or have worked in the past for banks or loan institutions. The percentage of appraisers acting in the market on behalf of mortgage lenders is somewhere around 90%. This number may not be statistically significant as it is the byproduct of a survey made by the authors but still it points toward a market in which home price estimates are being dictated by appraisers employed by lending institutions. The evidence up to this moment indicates very little will from the body of appraisers to reduce value estimates and when we couple this with the coercive feedback we get an idea on how flexible price estimates can be.

The real-estate market is by itself a highly inefficient market with large informational asymmetries and a distinct sluggishness of both demand and offer. This distinct feature implies ultimately an asset price which does not reflect present market conditions, let alone future. The demand cycle usually leads the offer with a lag of even up to several years [19]. Office stocks in the US took long time to be assimilated by the market in the mid 1980's; the construction boom continued even when vacancy rates where increasing (clear sign that the market was already saturated). The real estate resembled an overweight bear unaffected by the occasional energetic fly coming to "arbitrage" the remaining honey on his paws. This market started to be very dynamic (in terms of trading) in the late 1990's. In only a couple of years flipping (buying a property and reselling for a fast profit) became a common thing. The market metamorphosed in a buzzing flock of hungry flies. Such a structural change would be possible perhaps only under some fundamental market or regulatory changes and to our knowledge none of these have been reported in the literature. The increased attractiveness of the real estate market might have been caused by the overlapping between a market at peak (attracting more investors) cheaper borrowing (allowing more house purchases) and a spill-over effect from the .com burst. At the time the stock market deflated, the capital and the speculators were looking for new markets in which to act and Shiller hints that these investors moved from the stock market to the real estate market [16]. This may have caused the high prices to get even higher and the self-enforcing effect to start inflating a potential real estate bubble (flipping occurred 3-4 times in a year with none of the transactions being conducted by a stable house inhabitant [16]).

3 The Model

3.1 Microeconomic determinants

We look at the situation in which a generic agent is indifferent between buying and renting a house when the agent needs to take a loan for the house purchase. The agent can either be buying for his own personal use or as an investment. We compare these two decisions in terms of utilities and find a functional relationship between the mortgage size and the interest paid so as to understand the market and the financing conditions under which the agent will derive more utility from buying.

Before looking at the agent's decision of buying or renting, we determine first the necessary condition that motivates the agent to borrow money in the first place. This condition will prove to be very helpful in the further development of the model as it will indicate us the correct path to follow. An agent would borrow money from the bank only if the interest rate is in such a relation to the agent's time-preference parameter that a wealth transfer increases the utility of consumption. Let us denote by δ the agent's subjective time-preference parameter while i will be the interest paid on the loan amount L. $U(\cdot)$ will be the utility function of the agent which for the moment being we assume to be linear[1]. We look at a one period model in which at t_0 the agent decides whether or not to take the loan based on the t_1 expected house price and the existing rental levels where the decision to borrow is based on the relation between i and δ. The agent will be willing to borrow if the utility from borrowing is higher than the utility from not borrowing:

$$U_b(C_0) + \frac{1}{1+\delta}U_b(C_1) \;\geq\; U_{nb}(C_0) + \frac{1}{1+\delta}U_{nb}(C_1) \tag{1}$$

where (C_0, C_1) represent the consumption levels at t_0 and t_1 and the indices b and nb indicate whether or not the agent borrowed. We assume the t_0, t_1 endowments are known and are denoted as W_0, W_1 while the prices of consumption are q_0, q_1 (we will assume there is inflation of consumption $q_1 > q_0$). The budget constraints of the individual are:

$$\begin{cases} q_0 C_0 & \leq & W_0 + L \\ q_1 C_1 & \leq & W_1 - (1+i)L \end{cases}$$

[1] Model will be extended by considering a logarithmic utility

Assuming binding constraints and replacing in (1) we obtain the following equivalent inequality:

$$\frac{W_0 + L}{q_0} + \frac{1}{1+\delta}\frac{W_1 - (1+i)L}{q_1} \;\geq\; \frac{W_0}{q_0} + \frac{1}{1+\delta}\frac{W_1}{q_1} \tag{2}$$

We solve for i to obtain the maximum acceptable level of interest that the agent is willing to pay on a loan:

$$i \;\leq\; \frac{q_1}{q_0}(1+\delta) - 1 := i_{max} \tag{3}$$

Therefore an agent would take a loan if and only if the interest rate satisfies (3). We assume that this interest will be such that it covers the inflation of consumption ($1 + i > q_1/q_0$). At the same time, the agent decides between the possibility of purchasing a house or renting a house. If he decides to buy, we assume that the house price is larger than his t_0 endowment and thus he will need to take a loan from the bank. The agent's decision to buy is based on a similar utility comparison:

$$\max_{\theta \in \{0,1\}} U(C_0) + \frac{1}{1+\delta}U(C_1)$$

$$\text{subject to} \begin{cases} q_0 C_0 + R\theta + (P_0 - L)(1-\theta) & \leq \; W_0 \\ q_1 C_1 + R\theta + (1+i)L(1-\theta) & \leq \; W_1 + (1-\theta)P_1^e \end{cases} \tag{4}$$

where P_0, P_1^e represent the house price at t_0 and the expected house price at t_1 respectively. If the agent decides to rent (i.e. $\theta = 1$) then his utility from renting will be:

$$U\left(\frac{W_0 - R}{q_0}\right) + \frac{1}{1+\delta}U\left(\frac{W_1 - R}{q_1}\right) = U_{rent} \tag{5}$$

If the agent decides to buy (when θ=0) then his utility function becomes:

$$U\left(\frac{W_0 - (P_0 - L)}{q_0}\right) + \frac{1}{1+\delta}U\left(\frac{W_1 + P_1^e - (1+i)L}{q_1}\right) = U_{buy} \tag{6}$$

The requirement of positive consumption from (6) will yield the minimum and maximum loan amounts the agent can borrow:

$$\begin{cases} L & \geq \; P_0 - W_0 := \underline{L} \\ L & \leq \; \frac{W_1 + P_1^e}{1+i} := \overline{L} \end{cases} \tag{7}$$

When binding, the first constraint in (7) on L is the mathematical equivalent of our initial assumption that the agent cannot purchase a house using only his initial endowment. When not binding, this constraint says that the agent can borrow at t_0 more than needed for the house purchase; this will imply increased consumption at t_0 at the expense of reduced consumption at t_1. We allow for this condition so that we can also incorporate in our model the possibility of home-equity tapping (taking a loan for personal consumption with collateral being the house). This assumption is motivated by the increased amounts and frequency of equity extraction. The modeling framework with just one period is not very realistic for the home-equity tapping because we would need to know the exact house value at t_1, nevertheless one assumption that we use is that both the agent and the bank share the same belief about the direction of P_1^e therefore when the agent will want to consume in the present against a future house price increase, the bank will agree because of common future price expectations. The agent will be just indifferent between buying and renting when

$$U_{rent} = U_{buy} \tag{8}$$

thus expanding (8) we obtain

$$\begin{aligned} \frac{W_0 - R}{q_0} + \frac{1}{1+\delta}\frac{W_1 - R}{q_1} &= \frac{W_0 - P_0 + L}{q_0} + \frac{1}{1+\delta}\frac{W_1 + P_1^e - (1+i)L}{q_1} \\ \frac{P_0 - R}{q_0} - \frac{1}{1+\delta}\frac{P_1^e + R}{q_1} &= \frac{L}{q_0} - \frac{(1+i)L}{q_1(1+\delta)} \end{aligned} \tag{9}$$

Solving for L as a function of i we obtain a functional relationship between the the loan amount and the interest paid on this loan:

$$L(i) = \underbrace{\left(\frac{P_0 - R}{q_0} - \frac{1}{1+\delta} \cdot \frac{P_1^e + R}{q_1}\right)}_{T_1} \cdot \underbrace{\frac{q_0 q_1 (1+\delta)}{q_1(1+\delta) - q_0(1+i)}}_{T_2} \tag{10}$$

Equation (10) gives those loan levels for which the agent is just indifferent between buying and renting under the specified market conditions (i.e. for a given set of $q_0, q_1, R, W_0, W_1, i, P_0, \text{and} P_1^e$). As soon as the loan amount is larger than the RHS of (10) than the agent is more willing to buy. For the moment being, the parameter we focus on is the interest rate i. This parameter is decisive in the development of the mortgage market therefore we will look for those pairs of (L, i) such that the agent borrows in order to buy the house and the bank will maximize its profit.

We are interested in the shape of the indifference curve describing those pairs of (L, i) such that $U_{rent} = U_{buy}$. For this purpose we look at the derivative of $L(i)$ w.r.t. i

$$\frac{\partial L}{\partial i} = \underbrace{\left(\frac{P_0 - R}{q_0} - \frac{1}{1+\delta} \cdot \frac{P_1^e + R}{q_1}\right)}_{T_1} \cdot \frac{q_0^2 q_1(1+\delta)}{[q_1(1+\delta) - q_0(1+i)]^2}$$

We require that L be a positive quantity which implies that we can have either $(T_1, T_2) > (0,0)$ or $(T_1, T_2) < (0,0)$. Yet recall condition (3) which was giving the maximum acceptable i. In this case condition (3) implies that $q_1(1+\delta) - q_0(1+i) > 0$ and as $\delta > -1$ (or else this would imply that $1/(1+\delta) < 0 \Leftrightarrow$negative utility from consumption at t_1) we see that T_2 will always be positive. But then this implies that T_1 will have to be positive also. Therefore the derivative of L w.r.t. i will be a positive quantity (thus L(i) is a convex function in i). Figure 1 represents the set of optimal loan amounts and interest rates together with the isoprofit lines of the bank (where $\pi'' > \pi' > \pi$ represent the banks isoprofit lines).

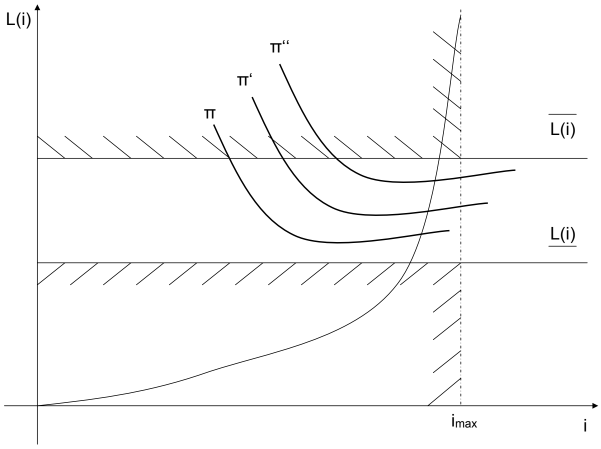

Figure 1: The indifference curve of the borrowing agent and the isoprofit lines of the bank

Equation (8) rewritten as an inequality (i.e. $U_{buy} > U_{rent}$) gives us a condition that the expected price has to satisfy so as to make the client more willing to buy:

$$U_{buy} > U_{rent} \;\Leftrightarrow\; \frac{L}{q_0} - \frac{(1+i)L}{q_1(1+\delta)} > \frac{P_0 - R}{q_0} - \frac{1}{1+\delta}\frac{P_1^e + R}{q_1} \tag{11}$$

which implies that

$$P_1^e > (1+\delta)q_1 \left\{ \frac{P_0 - R}{q_0} - \frac{L\left[q_1(1+\delta) - q_0(1+i)\right]}{q_0 q_1 (1+\delta)} \right\} - R \tag{12}$$

or equivalently

$$i \leq \underbrace{\frac{Lq_1(1+\delta) - T_1 q_1 q_0 (1+\delta)}{Lq_0} - 1}_{i_{exp}} \tag{13}$$

for

$$T_1 = \left(\frac{P_0 - R}{q_0} - \frac{1}{1+\delta} \cdot \frac{P_1^e + R}{q_1} \right)$$

When the price expectation P_1^e or the interest rate i satisfy the above relation then the agent will be more willing to buy.

3.2 The bank's profit-maximizing problem

The profit of the bank is given as the difference between cash inflows and cash outflows: if we assume that the amount deposited is equal to the amount lent then the bank's profit is given by $\pi = L(1+i) - L(1+j)$ (assuming zero costs). We assume that j is exogenously fixed. The bank's objective will be to maximize profit:

$$\max_i L(i-j)$$

$$\text{subject to} \begin{cases} i \geq 0 \\ i \geq j \\ L(i) \geq P_0 - W_0 := \underline{L} \\ L(i) \leq \frac{P_1 + W_1}{1+i} := \overline{L} \\ i \leq i_{exp} \end{cases} \tag{14}$$

The third and fourth constraints in the banks maximization problem translate into the following constraints on i:

$$L(i) \geq P_0 - W_0 \;\Leftrightarrow\; i \geq \frac{(P_0 - W_0)q_1(1+\delta) - T_1 q_1 q_0 (1+\delta)}{P_0 - W_0} - 1,$$

$$L(i) \leq \frac{P_1^e + W_1}{1+i} \;\Leftrightarrow\; i \leq \frac{q_1(1+\delta)(P_1^e + W_1)}{T_1 q_1 q_0 (1+\delta) + q_0(P_1^e + W_1)} - 1$$

The Lagrangian of this problem is:

$$\mathcal{L} = L(i)(i-j)-\lambda_1(-i)-\lambda_2(j-i)-\lambda_3(P_0-W_0-L(i))-\lambda_4\Big(L(i)-\frac{P_1^e+W_1}{1+i}\Big)-\lambda_5(U_{buy}-U_{rent})$$

For ease of readability we keep the conditions 3 and 4 as defined in the maximization problem, although when computing we'll use the actual constraints on i. The Kuhn-Tucker necessary conditions are

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial i} = 0 \\ \lambda_1 \geq 0 \ (= 0 \text{ if } i^* > 0) \\ \lambda_2 \geq 0 \ (= 0 \text{ if } i^* > j) \\ \lambda_3 \geq 0 \ (= 0 \text{ if } L(i^*) > P_0 - W_0) \\ \lambda_4 \geq 0 \ (= 0 \text{ if } L(i^*) < \frac{P_1^e+W_1}{1+i})) \\ \lambda_5 \geq 0 \ (= 0 \text{ if } U_{buy} > U_{rent}) \end{cases}$$

We will look at all the above conditions as they refer to different types of market competition; this will give us more insight in understanding the impact of the market situation on the present model. Before trying to find the admissible solution to the above optimization problem, a close scrutiny of the K.-T. conditions will allow us to simplify the problem. For the bank to be able to function we need $i^* > 0$; this strict inequality implies that $\lambda_1 = 0$. If we have a monopolist bank, then the bank will choose i^* such that $i^* = \min\{i_{max}, i_{exp}\}$ where recall that i_{max} is the maximum interest rate the agent would accept to pay on a generic loan and i_{exp} as given in (13) is the maximum interest for which the agent derives more utility from buying than from renting when a loan is needed for the house purchase. As soon i^* is chosen the loan amount can be derived and it will be either a boundary solution if $i^* = i_{max}$ or it will be given by $L(i_{exp})$.

If $L(i) < (P_1^e + W_1)/(1+i)$ then $\lambda_4 = 0$. We are interested mostly in the case where equity-tapping is present (as indicated by the empirical evidence) therefore we set $\lambda_3 = 0$. The Lagrangian for the perfect and imperfect competition is then:

$$\mathcal{L} = L(i)(i-j) - \lambda_2(j-i) - \lambda_5(U_{buy} - U_{rent})$$

with the FOC given by

$$L'(i)(i-j) + L(i) + \lambda_2 - \lambda_5[q_0 L(i) + q_0(1+i)L'(i) - q_1(1+\delta)L'(i)] = 0 \quad (15)$$

In the case of perfect competition ($i^* = j \Rightarrow \lambda_2 > 0$) the FOC becomes

$$L(i) + \lambda_2 - \lambda_5[q_0 L(i) + q_0(1+i)L'(i) - q_1(1+\delta)L'(i)] = 0 \qquad (16)$$

If i and P_1^e are such that the agent gets more utility from buying than from renting (which is the scenario we are interested in) then the constraint is not binding and so $\lambda_5 = 0$. But then (16) implies that $\lambda_2 = -L(i)$; as $L(i)$ is positive, it implies that $\lambda_2 < 0$. Therefore, in a perfect competition banking system where equity tapping is allowed and used and the housing market is in such a relation to the the mortgage rate and the rent that more utility is derived from buying than from renting, the bank will not maximize profit.

In the imperfect competition case the second constraint is not binding ($i^* > j \Rightarrow \lambda_2 = 0$); if we allow again for home-equity tapping and suppose there is an increased interest in home buying then (15) becomes

$$L'(i)(i-j) + L(i) = 0 \tag{17}$$

which is the condition for a free-solution. Solving for i we get:

$$\begin{aligned} T_1 \cdot \frac{q_0^2 q_1 (1+\delta)}{[q_1(1+\delta) - q_0(1+i)]^2} \cdot (i-j) + T_1 \cdot \frac{q_0 q_1 (1+\delta)}{q_1(1+\delta) - q_0(1+i)} &= 0 \\ T_1 q_1 q_0^2 (1+\delta)(i-j) + T_1 q_1 q_0 (1+\delta)\left[q_1(1+\delta) - q_0(1+j)\right] &= 0 \\ q_0(i-j) + \left[q_1(1+\delta) - q_0(1+j)\right] &= 0 \\ j &= \frac{q_1}{q_0}(1+\delta) - 1 \quad (18) \end{aligned}$$

Yet (18) is nothing else but the minimum level of interest received on a deposit that any agent would accept in order to lend money to a bank. It is obtained by equating (as previously done for the agent taking a mortgage) the utility resulting from depositing to the utility obtained from not depositing:

$$U_d(C_0) + \frac{1}{1+\delta} U_d(C_1) \ \geq \ U_{nd}(C_0) + \frac{1}{1+\delta} U_{nd}(C_1) \tag{19}$$

where the budget constraints of the individual are:

$$\begin{cases} q_0 C_0 & \leq & W_0 - L \\ q_1 C_1 & \leq & W_1 + (1+j)L \end{cases}$$

Thus in an imperfect competition the bank maximizes profit only when it pays the minimum acceptable rate on deposits (minimum acceptable from the point of view of the depositing agent).

To our purpose, we will assume that the bank acts in an environment of imperfect competition where the interest rate is determined by the interaction between the competing banks and in our model is set so as to satisfy the client's constraints. In this case, given a level of interest paid to the bank (where i will be the result of the interdependence between the competing financial institutions), the bank will maximize profits only when the rate paid on deposits is given by (18).

3.3 The dynamics of the real-estate market

Recall inequality (12). When this relation holds, the utility from buying is higher than the utility from renting. In our model this will lead to an increased demand for housing which in turn will lead to increasing house prices. This assumption makes sense because the demand cycle leads the offer cycle by a period of 12 to 15 months (which represents the average necessary time needed to complete a new construction - from license to delivery) therefore an increased demand will push and keep the prices up for some time before offer can balance the excess. Recall that the agents willing to buy a house will do this due to the increased utility therefore the increased demand will represent both demand for shelter as well as demand for the investment asset. We assume that all the agents in our model share the same type of expectation about the future price, where the conditional expected price is given by:

$$\mathbb{E}_t[P_{t+1}] = P_t + \tilde{a}(\bar{p} - P_t) + \tilde{b}(P_t - P_{t-1})$$

where $\mathbb{E}_t[\cdot]$ is the expectation conditional on time t information, $\tilde{a} > 0$ is the degree of fundamentalism, $\tilde{b} > 0$ is the degree of trend-chasing and $\bar{p}$ is the stationary solution (this is the value that we assume the price will stabilize to). Rewriting (12) we get a condition that today's price has to satisfy so as to induce house purchasing

$$P_t < \frac{P^e_{t+1} + R}{(1+\delta)a} + \frac{L[(1+\delta)a - (1+i)]}{(1+\delta)a} + R \tag{20}$$

When this condition is satisfied the mortgaging agent would be more willing to buy thus as soon as the difference between the LHS and the RHS is positive our model predicts an increasing price in the future. We assume here that the loan amount is given by $L = (P^e_{t+1} + W)/(1+i)$. Denote $a = q_{t+1}/q_t$ - we set a constant inflation rate for each consecutive period. To be in line with the empirical evidence we fix the wealth W so as to account for the increasing debt-to-income ratio. In the first stage, in order to simplify the dynamics problem we set i so as to satisfy the conditions from the previous subsection. We assume that the actual price dynamics will be given by

$$\begin{aligned} P_{t+1} &= P_t + d(P^*_t - P_t) \\ P^*_t &:= \frac{P^e_{t+1} + R}{(1+\delta)a} + \frac{L[(1+\delta)a - (1+i)]}{(1+\delta)a} + R \end{aligned} \tag{21}$$

A short digression on the matter of the interest rate i is needed here. We know the minimum loan amount for which the agent is just indifferent between buying and renting. This quantity is given by (10). If we know that the loan amount approved by the bank will be

$$L(i) = \frac{P^e_{t+1} + W}{1+i}$$

then we can obtain the level of the interest rate by maximizing profit w.r.t. to i subject to the bank reservation price. We would then get a different interest i for each period as i depends on P^e_{t+1} and this quantity will vary with time. For simplicity we compute the first period's interest and use it for the subsequent periods also.

As long as (20) holds, the increased utility from buying implies increased demand thus an upward push on tomorrow's price where d will be a parameter driving the actual price increase (we will call it generically the 'bubble parameter'). Replacing in (21) for L and P^e_{t+1} we obtain

$$P_{t+1} = P_t + d\left\{\frac{P^e_{t+1} + R + \frac{P^e_{t+1}+W}{1+i}[(1+\delta)a - (1+i)]}{(1+\delta)a} + R - P_t\right\} \tag{22}$$

and solving the above second-order linear difference equation we obtain the following stationary solution

$$\bar{p} = \frac{R[1+(1+\delta)a](1+i) + W[(1+\delta)a - (1+i)]}{(1+\delta)\, a\, i} \tag{23}$$

For the above difference equation to converge, the parameters $\tilde{a}, \tilde{b}$, and d will have to satisfy some particular conditions. These conditions are that the characteristic roots of the characteristic equation will have to be less than one in absolute value (they have to belong to the unit circle). Replacing in (22) for P^e_{t+1} and grouping terms find the following characteristic equation

$$r^2 - \left[(1-d) + \frac{d}{1+i}(1-\tilde{a}+\tilde{b})\right] r + \frac{\tilde{b}d}{1+i} = 0$$

The conditions of convergence[2] are that the roots of this quadratic equation be less than one: $|r_1| < 1$ and $|r_2| < 1$. Solving this system of inequalities leads to

$$\begin{aligned} \tilde{b}d - (1+i) &< 0 \\ -\frac{[(1-d) + (d/(1+i))(1-\tilde{a}+\tilde{b})]}{2\sqrt{\tilde{b}d/(1+i)}} + 1 &\geq 0 \\ -\frac{[(1-d) + (d/(1+i))(1-\tilde{a}+\tilde{b})]}{2\sqrt{\tilde{b}d/(1+i)}} - 1 &\leq 0 \end{aligned}$$

[2]We require that the system presents oscillations around the equilibrium value

A numeric implementation shows the stability of the above solution (Figure 2). The value of the parameters used to obtain the path in Figure (2) are given in the table below:

δ	i	R	W	$\tilde{a}$	$\tilde{b}$	d
0.31	0.13	31	190	1	0.5	1.1

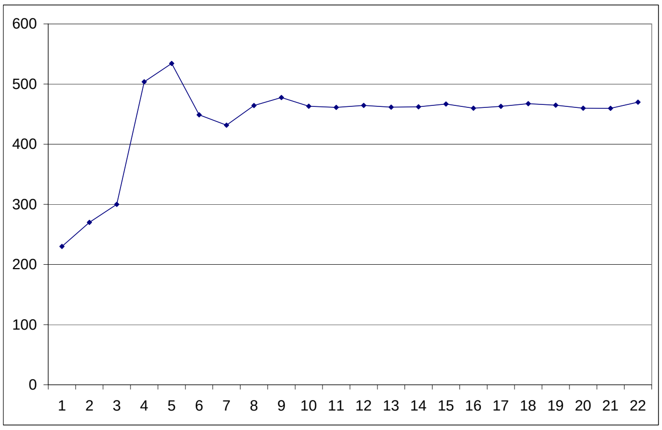

Figure 2: The stationary level is reached as soon as the fundamentalists' influence becomes dominating

It is of great interest to see how the stationary solution behaves under different values for the above market parameters. A brief comparative static analysis shows that the derivative of $\bar{p}$ w.r.t. to both R and W is increasing - larger values of the rent or of the endowment imply a higher actual price. The higher will the rent level be the more people would resort to buying instead of renting thus pushing the prices up. This can be seen also from equation (20) - larger values of R imply a higher difference between P_t and P_t^* which will reflect in a higher actual price for the next period. The derivative of $\bar{p}$ with respect to i is decreasing: a higher loan interest will cause a smaller value for the stationary solution (thus smaller actual prices). The mechanics of this opposite movement can understood by recalling that $L(i) = (P_{t+1}^e + W)/(1+i)$ - higher interest rate values decrease the loan value which implies that the loan value will play a smaller role in (20) thus decreasing the difference $P_t^* - P_t$. This feature of the model comes to support the traditional view that increasing (decreasing) interest rates lead to decreased (increased) interest in house purchasing.

Up to this moment we have not yet explicitly included in our model the effect that the upward revising bias or the focus on inflation have on the loan amount. The previously cited empirical work in the field of behavioral real-estate (recall ([14]) indicates that when given feedback on their evaluations, appraisers are more likely to update their estimates upwards than downwards; their tendency is to overestimate and almost never underestimate the future house price. Also, appraisers look at inflation as being one of the most important parameters determining the house price. In our case this asymmetric behavior and the focus on inflation are captured by the following dynamics of the loan amount

$$L_{t+1} = L_t + \underbrace{\left(1 - \frac{1}{e^t}\right)}_{f_u}(P_t - P_t^e)\mathbb{I}_{\{P_t > P_t^e\}} + \underbrace{P_t(a-1)}_{f_d}\mathbb{I}_{\{P_t < P_t^e\}} \tag{24}$$

This equation relates L_{t+1} the loan amount issued by the bank for the period t to $t+1$ to the previously issued amount L_t for the period $t-1$ to t. Equation (24) tells us that if the appraiser observes at t that his price estimate P_t^e was below the actual price ($P_t > P_t^e$) then he would be willing to increase the loan amount for the next period with a fraction of the difference between the actual price and his estimate. The appraiser's focus on inflation as the main determinant of the future price will dictate the level to which he will compare the actual price: if the actual price turned out to be higher than the one-period before price multiplied by the q_{t+1}/q_t than the appraiser will be increasing the loan amount by a fraction f_u of the difference - the more there are consecutive periods in which $P_{t+1} > a\, P_t$ the higher the fraction of the difference that is included in the next period's loan. This retrospective analysis (comparing the actual market price to common estimates) gives the appraiser evidence whether or not his estimate was accurate; if the price increase was underestimated, the bank has suffered a loss from issuing a loan smaller than what it could have issued and what the agent would have accepted (assuming the maximum acceptable both for the bank and for the agent is given as before by $L_t = (P_{t+1}^e + W)/(1+i)$). The upward revising bias is incorporated in our analysis through the choice of the two weighting functions: f_u the fraction of the price difference incorporated when feedback indicated a too small price expectation as compared to the market and f_d the fraction of the price difference when the opposite is observed. Due to the choice of f_u when $P_t > P_t^e$ for several sequential periods than the appraisers will be incorporating an ever increasing amount of the price difference $P_t - P_t^e$. If the opposite happens than the appraiser will increase the loan amount only by the inflation rate $a-1$ times the price from that period. In this setting, we assume that the retrospective expected price to which the appraiser compares the actual price will only have to possible evolutions: it will increase either more than the inflation rate or less than the inflation rate but it will never decrease. Figure 3 shows the above model when the biases are present as compared to the model without biases. Although a strong assumption a quick look at the actual history of the real

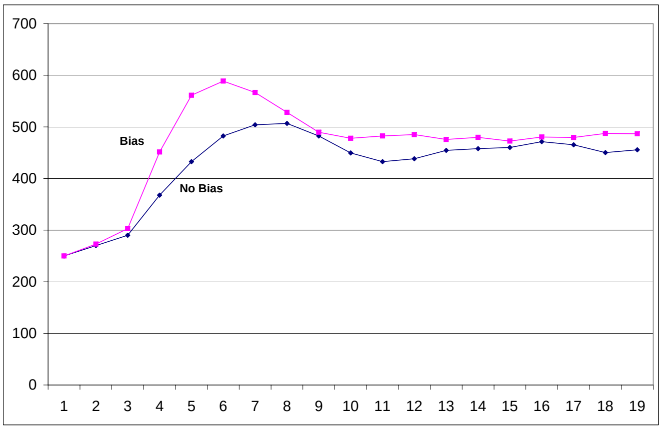

Figure 3: The presence of biases changes the actual price development

estate market reveals that since the beginning of the century prices declined sporadically and only for very brief periods therefore assuming that house prices would increase with inflation is not an extreme assumption for the appraiser to make. Three qualitative features can be readily observed in the model featuring biases: the actual price increases more as compared to the situation when biases are not present, the variability of the price is smaller on the long run (supporting thus the empirical evidence of appraisal smoothing) and the price tends to its stationary level slower than in the case of no biases.

3.4 Conclusions

Although many oversimplifying conclusions were used to obtain a tractable model, the main message is that the presence of a large body of agents acting under the influence of cognitive biases can alter the natural evolution of market values. The model predicts that if a bubble is created and the above mentioned biases are present then the prices will increase more than under normal conditions, they stabilize to their long-run values slower and will have the long-run tendency to vary less.

3.5 Outlook

The next steps to take are to calibrate the model to actual data in order do obtain estimate values of the parameters of interest. Some essential facts would deserves further attention: the evolution of the rents should be explicitly modeled, the competition between the lending institutions may change the existing conditions for the interest rates and this should be accounted for; heterogeneous beliefs about the price development could offer more insight into the dynamics of the market.

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